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# Periodicity of world crude oil maritime transportation: Case analysis of Aframax Tanker market



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#### ABSTRACT

The oil tanker shipping market always presents periodicity following varied laws in different periods. However, most of the studies on the tanker freight rate market are short-term projections, while those on its long-term periodicity characteristics are mostly qualitative ones. In order to study the periodic variation law of the tanker market, this paper uses quantitative methods to figure out the cycle duration and amplitude of different scales of Aframax tanker's freight, and predicts the long-term variation trend of freight rate on that basis. This paper selects the one-year charter freight rate of Aframax tankers (110,000 DWT D/H) to comprehensively analyze the Aframax tanker freight market variation law, and further divides the cycle into three major categories. With the help of wavelet analysis, this paper quantifies the periodic volatility of the Aframax tanker shipping market, and figures out a quarterly cycle of 11.2 months, a short-term cycle of 3.7 years, and a medium-to-long-term cycle of 11.9 years. Based on the cycle characteristics, the paper predicts the market cycle trend to the next medium-tolong-term cycle and further to the year 2030, so as to offer some reference to oil tanker shipping market players for better decision-making.

## 1. Introduction

Crude oil, as an important source of energy, has been regarded as a crucial input in the process of economic growth [[1](#page-7-0)]. As the derived demand for international trade, the crude oil marine shipping market both serves international trade and reflects the development and tendency of the international economic environment [[2](#page-7-1)]. Out of energy cost considerations, there have been many studies on the future trend of the crude oil shipping freight rate and its dynamic and volatile characteristics [3–[5\]](#page-7-2). However, these predictions seldom took into account the periodic characteristics of the tanker freight rate and the implications of many complex factors on the tanker freight rate such as the crude oil price and shipbuilding price. There have been few studies on quantifying the periodicity of tanker freight rate of different time scales [[6](#page-8-0),[7](#page-8-1)].

During the 2008 financial crisis, the international crude oil price plummeted by 73% through a short period of five months, from the high of \$150/barrel in July 2008 to less than \$40/barrel in December 2008. The international crude oil price remained below \$70/barrel in March 2018. The three oil crises in history have had a huge impact on the supply and demand as well as the prices of international oil products, and the impact was further carried to the tanker freight market. The adjustment of market supply and demand on the time scale is exhibited in the form of periodic fluctuations of tanker freight rate [[8](#page-8-2)]. Meanwhile, considering the weak recovery of the world economy in recent years coupled with a large number of new tankers built or ordered before the outbreak of the international financial crisis, excessive shipping capacity supply resulted in a significant depression in the tanker freight rate [\[3\]](#page-7-2). Enterprises closely related to tankers, such as shipyards and ship companies, will make tanker charter as well as

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tanker manufacturing and dismantling decisions based on changes in tanker freight rate. Therefore, the importance for entrepreneurs and policymakers to make an accurate forecast of tanker freight volatility and dynamicity cannot be exaggerated.

International tankers can be divided into six categories by deadweight: general-purpose, Handymax, Panamax, Aframax, Suezmax tankers and supertankers. Aframax ship is designed based on the Average Freight Rate Assessment (AFRA), with ship deadweight ton design based on the best revenue point of freight income and cost. Therefore, the Aframax ship is also called "freight ship". Because of draught restrictions of port facilities, the ports of many oil-exporting countries find themselves hard to receive Very Large Crude Carriers (VLCCs), while Aframax tankers are widely used in the Black Sea basin, the North Sea, the Caribbean, Chinese sea areas and the Mediterranean Sea. For this reason, the international tanker market has a great demand for Aframax vessels. In view of the versatility and economical efficiency of this ship type, this paper chooses Aframax tanker freight rate to study the cycle characteristics of the tanker freight rate market.

A significant majority of research focuses on forecasting the freight rate trend. Many prediction techniques, such as statistical regression model, represented by ARIMA and ARCH [[9](#page-8-3)], heuristic algorithm, represented by neural network  $[10,11]$  $[10,11]$  $[10,11]$  and genetic algorithm  $[4]$  $[4]$  $[4]$  have been used to forecast the shipping market. Munim & Schramm [[9](#page-8-3)] forecast the container shipping freight rate with the ARIM-ARCH model, with the results providing comparatively better outcome than existing freight rate forecasting models, performing short-term forecasts on a weekly and monthly basis. However, the ARIM-ARCH model is not suitable for long-term forecasting as freight rate exhibits multiple periodicity features. von Spreckelsen et al. [[12\]](#page-8-6). investigated the performance of linear and non-linear prediction methods for tanker freight rate, and found that non-linear methods such as neural networks are suitable for short-term forecasting and trading of the freight rate. Nonetheless, most of the aforementioned forecasting methods are applicable to short-term forecasts of freight rate, and related studies on the long-term trend and periodicity characteristics of the market are necessary to make long-term forecasts.

Some researchers attempted to understand the time-varying characteristics of freight rate volatility by focusing on some of its influence factors [\[13](#page-8-7)]. Other researchers have attended to the correlation and influence mechanisms between markets and influence factors [\[14](#page-8-8)–16]. Chen et al. [[2](#page-7-1)] studied the multifractal cross-correlations between the crude oil price and tanker freight rate employing the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), and they found that the strength of multifractality after the financial crisis was larger than that before. The periodic characteristics were also considered for predicting market tendency. Kavussanos and Alizadeh-M [\[17](#page-8-9)] investigated the seasonality of dry bulk freight rate, measuring and comparing it across freight rates of different vessel sizes. Their results illustrated that spot rates for larger vessels exhibited higher seasonal fluctuations than those for smaller ones, and they attributed asymmetries in seasonal fluctuations in freight rates over different market conditions to the high and low elasticities of supply. Sun et al. [\[6\]](#page-8-0) studied the multiscale correlation between freight rates and oil prices based on the ensemble empirical mode decomposition model, the results showed that tanker freight rates and oil prices exhibit different multiscale properties and were significantly correlated in the medium and long term. Despite the research on tanker freight volatility and forecasts in many studies, the time-varying, non-linear and local non-stationary features of freight rate make modeling of its inherent dynamics a challenging task.

When studying periodic characteristics of the shipping market, Stopford [\[18](#page-8-10)] divided the shipping market cycle into four phases: the trough phase, the recovery phase, the crest phase and the decline phase. The results proposed three kinds of cycles in the shipping market: a long cycle (60 years), a short cycle (5–10 years), and a seasonal cycle (less than 1 year). However, these cycle durations were not worked out using quantitative methods. Gkochari [\[19](#page-8-11)] analyzed a dataset of the Capesize

shipping market based on option games that helped to explain the existence of boom-and-bust cycles in shipping. Goulielmos and Siropoulou [[20\]](#page-8-12) used the rescale range analysis method to calculate the duration of cycles manifested in the prices of second-hand tanker ships, finding the existence of two cycles of tanker ship prices with durations of four and eight years, respectively. Those results are, of course, preliminary; much more data is required to test their forecasting validity, and this should be regarded as just a starting point in this provocative area of research. Randers and Göluke [\[21](#page-8-13)] predicted the turning points in shipping freight rate and explained the world's shipping market cycles as balancing feedback loops: a capacity adjustment loop and a capacity utilization adjustment loop. Papailias et al. [[22\]](#page-8-14), for example, investigated the BDI with comprehensive forecasting performance evaluation methods, finding that there existed a strong pattern of cycle duration of between three and five years, and that this pattern was relatively stable across time. However, these results vary greatly due to the complexity of influencing factors in the shipping market. Yin and Shi [\[23](#page-8-15)] analyzed the seasonality patterns of container shipping freight rate and the results revealed that regular seasonality fluctuation patterns are within a one-year period. However, the study failed to look at the cycles on other time scales.

Despite a large number of studies on identifying and forecasting the influencing factors of tanker freight rate, there remains a dearth of precise quantitative studies, especially for the tanker freight cycle. Since empirical methods fail to meet these requirements, a quantitative study of periodic variations should be introduced to lend guidance to enterprise decision-making. This paper followed the practice of dividing the cycles by three different time scales in order to analyze the market periodic characteristics more precisely. We used wavelet analysis, which is particularly suitable for the time-frequency scale of data, to focus on features of tanker freight cycle (especially the cycle duration). We obtained the periodic characteristics of tanker freight rate from relevant wavelet coefficient curves to analyze the periodic characteristics of the original data. Then we reconstructed the wavelet coefficient curves with different variation features to characterize the periodic variation on three different time scales. Our contributions can be described as follows:

(1) Unlike those studies focusing on periodic characteristics on just one time scale, this paper divided the cycles by three typical time scales look into different cycles' features. This enables a more accurate way to describe the long-term periodic variation trend of Aframax tanker freight rate; (2) This paper adopts quantitative methods to calculate the duration and amplitude of Aframax tanker freight rate on varied time scales and forecasts the long-term trend of freight rate on that basis; (3) This paper establishes a theoretical basis and framework for analyzing periodic variation features of tanker freight rate using wavelet analysis and gives guidance to decisionmarking of tanker shippers and policy-makers.

The rest of the paper is organized as follows: Section [2](#page-1-0) describes the wavelet analysis method; Section [3](#page-3-0) introduces the data; Section [4](#page-3-1) discusses the wavelet analysis results and forecasts the periodic variation and tendency in the next medium-to-long-term cycle. The conclusion is provided in Section [5](#page-6-0).

## <span id="page-1-0"></span>2. Methodology

Wavelet analysis was proposed by Morlet et al. [\[24](#page-8-16)] and received wide application in time series data research such as geography, climate, medicine and finance [\[25](#page-8-17)]. Some traditional methods such as ARIMA and VAR models require data to be stable, with the residual signal being white ones [\[26](#page-8-18)]. When looking into the inherent complexity and mutability mix of original data, Li et al. [\[3\]](#page-7-2) used the decomposition hybrid approach to divide the original data into a series of relatively simple but meaningful components following the "decomposition and ensemble" principle. Wavelet analysis is capable of decomposing time series into trend, cycle and noise [\[27\]](#page-8-19), and thus we can use it to indicate the periodic variation features in the original data on different time and frequency scales so as to provide theoretical support for forecasting market variations.

Unlike the Fourier analysis which involves frequency analysis only, wavelet transform scales and translates the mother wavelet function to generate a series of derived wavelets and then uses the wavelets to translate and compare the to-be-analyzed signals on the time axis to work out the wavelet coefficient that represents the degree of similarity between a signal and a wavelet [\[28](#page-8-20)]. Wavelet analysis overcomes the shortcomings of Fourier analysis the transform window size of which fails to vary with frequency in a short term and lacks a discrete orthogonal basis. Therefore, wavelet analysis can be used to analyze the time domain and frequency domain of data [\[29](#page-8-21)]. Since the monthly freight rate of Aframax tankers is subject to the influences of periodic factors dominated by oil prices, which renders similar periodic fluctuations into freight rate, and random influences of other non-periodic factors, such as employee wages, port service charges and navigation fees, we need to separate the low-frequency layer representing the periodic characteristics in the raw data from the high-frequency layer representing the non-periodic characteristics when analyzing periodic characteristics of freight rate data. The features of wavelet analysis make it well fit the requirements.

Accordingly, with the help of wavelet analysis, we studied the periodic variation characteristics of Aframax tanker freight to reconstruct the low-frequency layers within three time-scale cycles.

#### 2.1. Wavelet analysis

+∞

The wavelet analysis is a kind of time-frequency analysis, which is developed based on the foundation of Fourier transform. Fourier transform could well decompose a function of time  $f(t)$  into its constituent frequencies and its formula is defined as:

$$
F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt
$$
\n(1)

In the formula,  $\omega$  is the frequency, t is time.  $F(\omega)$  is the Fourier transform of the time series function  $f(t)$ .  $e^{-iwt}$  is the complex variables functions.

A wavelet transform is the development and extension of the Fourier transform. However, unlike Fourier transform which only involves frequency-domain translation, wavelet transform can translate the time-domain and frequency-domain at the same time [\[30](#page-8-22)]. Wavelet analysis approximates the original function by scaling and translating the mother wavelet function. The mother wavelet function  $\Psi(x)$  is a square-integrable spatial function, x is the time variable, that is,  $\psi(x) \in L^2(R)$ .  $\hat{\psi}(\theta)$  stands for the Fourier transform form of  $\psi(x)$ ,  $\theta$  is the frequency,.  $\hat{\psi}(\theta)$  satisfies the following admissible conditions:

$$
0 < \int\limits_R \frac{|\hat{\psi}(\theta)|^2}{|\theta|} d\theta < \infty \tag{2}
$$

$$
\hat{\psi}(\theta = 0) = \int_{-\infty}^{+\infty} \psi(x) dx = 0
$$
\n(3)

The wavelet sequence from translating and scaling the mother wavelet is also called daughter wavelet., which could be defined as:

$$
\psi_{a,b}(x) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{x-b}{a}\right), \quad a, b \in \mathbb{R}, \ a \neq 0 \tag{4}
$$

In the formula,  $\psi_{a,b}(x)$  indicates to scale the wavelet function  $\psi(x)$ by a times and translate it by b units. a is the scale factor that scales the mother wavelet, a could stretch and shrink the mother wavelet in time. When  $|a| > 1$ , it indicates to scale up the mother wavelet, and when  $|a|$  < 1, it indicates to scale down the mother wavelet. *b* is the

translation factor that translates the mother wavelet. In this way, a could rescale the center frequency of the mother wavelet. When  $b > 0$ , it indicates to shift the mother wavelet right, and when  $b < 0$ , it indicates to shift the mother wavelet left. In this way, the wavelet transform can translate both time-domain and frequency-domain.

When  $a$ ,  $b$  are continuous values, the  $f(x)$  continuous wavelet transform (CWT) in  $L^2(R)$  is defined as:

$$
W_{x,\psi}f(a, b) = f(x), \psi_{a,b}^{*}(x) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(x) \psi^{*} \left(\frac{x-b}{a}\right) dx
$$
\n(5)

In the formula,  $W_{x,\psi}f(a, b)$  is the wavelet coefficient. With the formulas of CWT above, The wavelet analysis is a measurement of the similarity between the daughter wavelets and the origin signal function. The coefficients calculated indicate how close the function is to the daughter wavelet at that particular scale  $a.$  < >  $\Diamond$  denote the inner product.  $\psi^*$  is the complex conjugate of  $\psi$ .  $\langle f, \psi \rangle = (1/2\pi)\langle \hat{F}, \hat{\psi} \rangle$ , where  $\hat{F}$  is the Fourier transform of *f*. So CWT  $W_{f, \psi}(a, b)$  could be written as:

<span id="page-2-0"></span>
$$
W_{f,\psi}(a,\,b)=\frac{\sqrt{|a|}}{2\pi}\int_{-\infty}^{+\infty}\psi^*(a\theta)\hat{F}(\theta)e^{i\theta b}d\theta\tag{6}
$$

It should be noted that in Formula  $(6)$ ,  $b$  is the time dimension, which is a location parameter. *a* stands for the scale dimension, indicating that  $W_{f,\psi}(a, b)$  is the time-scale representation of  $f(x)$ . Meanwhile, we can conduct continuous wavelet analysis through vector centralization to represent the inverse relation between scale and fre-quency [[30\]](#page-8-22).

<span id="page-2-1"></span>The inverse wavelet transform of  $f$  is given by Daubechies  $[31]$  $[31]$  $[31]$ :

$$
f(b) = \frac{1}{C} \int_{-\infty}^{\infty} \int_{0}^{\infty} a^{-2} W_{x,\psi} f(a, x) \psi_{a,b}(b) da \ dx \tag{7}
$$

In the formula, C is a constant. [Formula \(7\)](#page-2-1) can be regarded as a process of reconstructing the original function  $f(x)$ , that is, when the wavelet transform  $W_{x, \psi} f(a, b)$  is worked out,  $f(b)$  can be regarded as superposition of the daughter wavelet  $\psi_{a,b}(x)$ .

When  $a$ ,  $b$  are uncontinuous values, we can obtain the discrete wavelet transform (DWT). When discretizing the scale and location parameters  $(a, b)$ ,  $a$  can be chosen as  $a_0^m$ , where  $m$  is an integer and  $a_0 > 1$ . Meanwhile, one can choose  $b = nb_0 a_0^m$ , where whether  $b_0 > 0$ depends upon  $\Psi(x)$  and *n* is an integer [\[32](#page-8-24)]. Then we define:

$$
\psi_{m,n}(x) = \frac{1}{\sqrt{a_0^m}} \psi \left( \frac{x - nb_0 a_0^m}{a_0^m} \right) = a_0^{-m/2} \psi (a_0^{-m} x - nb_0)
$$
\n(8)

When  $\Psi_{m,n}(x)$  is obtained, the DWT can be shown as:

$$
Wf(m, n) = a_0^{-m/2} \int f(x) \psi(a_0^{-m}x - nb_0) dx
$$
\n(9)

Wavelet analysis can obtain high- and low-frequency signal layers by changing the scale factor  $a$  and  $b$ . As shown in [Fig. 1,](#page-3-2) the original time series data is introduced in the data description part, and the wavelet decomposes the original signal S into a low-frequency layer  $\mathfrak{a}_n$ and a high-frequency layer  $d_n$ , that is,  $S = a_i + \sum_{i=1}^{i} d_i$  The original signal can be decomposed into a linear combination of wavelet coefficient function (scale function) and wavelet function. In this function, wavelet coefficient function generates low frequency part and wavelet function generates high frequency part. As the wavelet coefficient functions (wavelet coefficient curves) obtained by the wavelet analysis has the same frequency as the original signal, in this case, we could obtain the periodical features of the original data [[25\]](#page-8-17).

The choice of the wavelet function  $\Psi(x)$  is not unique or arbitrary. *Ψ*(*x*) should have unit energy, that is,  $\int |\Psi(x)|^2 dx = 1$ . Meanwhile, it should have compact support to be able to attenuate to the position in space fast enough. Besides, it has zero means, that is,  $\int \Psi(x) dx = 0$ . Therefore, there are many eligible wavelet functions available, such as Mexican hat, Haar wavelet and Daubechies wavelet [[31\]](#page-8-23). In usual

<span id="page-3-2"></span>

Fig. 1. Illustration of the wavelet decomposition process.

cases, scholars filter available wavelet functions based on the basic characteristics, and verify the validity of the selected wavelet function through experiments.

This paper chose Daubechies wavelet, which was proposed by Daubechies [\[31](#page-8-23)], as the mother wavelet function. dbN (N is the vanishing moment) has no definite functions except db1, which is the same as the Haar wavelet function which can be written as:

$$
\psi_H = \begin{cases}\n1, 0 \le x \le \frac{1}{2} \\
-1, \frac{1}{2} \le x \le 1 \\
0, \text{ else}\n\end{cases}
$$
\n(10)

Main characteristics of the dbN wavelet function include: (1) an effective support width of 2N-1; (2) a vanishing moment of N; (3) being asymmetrical, orthogonal, compact supported and biorthogonal.

## <span id="page-3-0"></span>3. Data description

In this paper, we selected standard Aframax tanker for research. This type of tanker has a small volume and is admissible at most ports. What's more, it is generally designed centering around the best revenue point of freight revenue and cost, hence representative of typical freight tankers in international tankers. As the modern Aframax tanker (110,000 DWT tanker from start April 2000) has took the place of the old Aframax tanker (95,000 DWT early 1990s built tanker this paper is based on the monthly one-year time charter freight rate data (\$/(Day), and get the per 10000 DWT freight rate (\$/(Day\*10000 DWT) of a total of 518 groups of Aframax tankers (95,000 DWT tanker from Jan 1976 to Mar 2000 and 110,000 DWT tanker from April 2000 to Feb 2019) in the database of Clarkson SIN, as shown in [Fig. 2.](#page-3-3)

#### <span id="page-3-1"></span>4. Results and discussion

#### 4.1. Selection of wavelet function for Aframax tankers

<span id="page-3-3"></span>Previous research unveils cycles on three time scales for the dry bulk carrier market [[18\]](#page-8-10). Many studies also pointed out that oil price exhibits periodicity [\[33](#page-8-25)]. The wavelet analysis used in this paper can help locate signals of both low-frequency layer and high-frequency layer of the tanker freight rate. The low-frequency layer is selected for analyzing

the volatility periodicity characteristics of the tanker freight rate. However, different wavelet functions may lead to different processing results. Main wavelet functions which have orthogonality and were currently used include Haar, symN, CoifN and dbN wavelet [\[34](#page-8-26)[,35](#page-8-27)]. In view of the time and frequency requirements of the cycles studied in this paper, we combined the reference and experimental data to select the wavelet function as there has no standard method to choose the wavelet function.

This paper follows two main criteria to choose the best wavelet, first, the low-frequency layer obtained after the wavelet transform is required to be smooth to a certain extent to facilitate calculation of the vibration cycle step size and could reflect variation features of the original data. Second, it might have boundary problem if the wavelet function's support length is too long, and it might cause low vanishing moment, which would cause the dispersion of signal energy, if the wavelet function's support length is too short. Hence, this paper chose the wavelet function whose support length is in the range of 5–9. During function determination experiment, the original signal was decomposed into 3 layers and vanishing moment N was 4, The low-frequency layer a3 was chosen for comparison and the results are shown in the [Fig. 3](#page-4-0) and [Table 1.](#page-4-1)

As shown in [Fig. 3,](#page-4-0) the result of db4 and coif4 could well reflect variation features of the original data. while this paper chose the db4 as it has more suitable support length as shown in [Table 1.](#page-4-1) As the vanishing moment N could choose from 1 to 6 (N larger than 6 might not satisfy the support length requirement), in a bid to identify the best vanishing moment N, this paper decomposed original signal with different dbN wavelet functions ( $N = 1$  to 6) to figure out the corresponding low-frequency layers. [Fig. 4](#page-5-0) shows the comparison of the lowfrequency layer a3 with the original signal layer. [Table 2](#page-5-1) shows the standard deviation (STD) results of different dbN wavelet functions in three low-frequency layers.

When the vanishing moment N of the wavelet function increases, the a3 low-frequency layer curve is smoother, and data stability is also improved significantly. As shown in [Fig. 4,](#page-5-0) db4's low-frequency layers after removing the high-frequency layers are smooth enough and can better reflect the periodic volatility of the Aframax Tanker Freight data. [Table 2](#page-5-1) shows db4 has better STD performance than any other dbN. In this case, this paper chose db4 as the mother wavelet.



Fig. 2. Aframax Tanker's freight per 10,000DWT (\$/Day∙10,000 DWT).

<span id="page-4-0"></span>

Fig. 3. Comparison of the a3 Layer After Different Wavelet Transform.

<span id="page-4-1"></span>Table 1 Performance of different wavelet functions.

Wavelet function	dh4	Haar	Coif4	Sym <sub>4</sub>
Standard deviation	100.00	155.72	102.58	120.88
Support length $(N = 4)$	$2N-1 = 7$		$6N-1 = 23$	$2N-1 = 7$

# 4.2. Forecasting Aframax tanker freight cycle duration based on onedimensional continuous wavelet analysis

One-dimensional continuous wavelet analysis on monthly freight rate data of Aframax tankers helps us get the wavelet coefficients and wavelet coefficients map is showed in [Fig. 5](#page-6-1). The map features typical light and dark interval variations, which shows the extreme values of the wavelet coefficient curves (WCCs) at different scales (ordinate), and the interval distance increase with the scale (ordinate) goes up, which manifests different cycle length of the Aframax tanker freight as the WCC has the same periodical feature as the original signal. This paper obtained the cycle lengths from the mean interval distances of WCC's extreme values (local maxima and local minima), and chose the cycles from the WCCs whose extreme values' distances have the stable values and low STD, the results are shown in [Fig. 6.](#page-6-2)

In [Fig. 6,](#page-6-2) we can see different interval's mean values in different scale ranges. There are scale ranges where the interval's mean values have huge variations, and they mean that the cycle lengths are interfered by different types of cycles. These parts are corresponding to the parts in [Fig. 5](#page-6-1) where the WCC's extreme values are interconnected, and these cycles are instable and should be excluded. Meanwhile, in the scales where intervals have stable mean values, the mean interval values calculated by the local maxima are larger than values calculated by the local minima, which means the cycles' wrests' distances are always larger the distances between the troughs. This characteristic is consistent with the phenomena of Aframax tanker freight market, whose cycles' depression and recovery periods are always larger than the boom and bust periods.

In this paper, the intervals with stable values and low STD within a certain range of scales were regarded as the typical cycle of the Aframax tanker freight, and it could be found that there have four ranges of scales where the intervals have stable mean values and low STD, and these intervals could be concluded into 3 typical cycles, namely the short-term cycle, the medium-to-long-term cycle, and the long-term cycle. The results are shown in [Table 3](#page-6-3).

By selecting the ordinate values with the smallest STD between interval distances in different scale values, we can get the corresponding cycle lengths and wavelet coefficient curve. As shown in [Table 3,](#page-6-3) in the short-term cycle range (scales from 31 to 45), the interval distance's STD of the wavelet coefficient curve on the 35 scale is the smallest. If we use the cycle value of the curve as the one for the short-term cycle range, we can work out the cycle of the wavelet coefficient curve is 41.2 months (or 3.45 years). In the same approach, we can work out the medium-to-long-term cycle values of 85.3 months (or 7.11 years) on the 65 scale and 107.6 months (or 8.97 years) on the 86 scale, and the long-term cycle value of 143.5 months (or 11.96 years) on the 147 scale.

## 4.3. Prediction and analysis of Aframax tanker freight rate cycle

This paper chose the corresponding WCCs to represent different cycles of Aframax tanker freight, we used the ARIMA method to extend the four wavelet coefficient curves, which have been normalized, from Mar. 2019 to Feb. 2032 according to the corresponding periodic characteristics to the next long-term cycle – the year 2032, and superimposed the wavelet coefficient curves on the four scales to get an accumulated wavelet coefficient curve that can represent and predict the volatility characteristics throughout the entire cycle, as shown in [Fig. 7.](#page-7-4)

- (1) The predicted short-term cycle of Aframax tanker freight is 41.2 months (or 3.45 years), which mirrors the short-term periodic fluctuations of the market. According to [Fig. 7](#page-7-4) (b), the Aframax tanker freight was on a decline in Feb 2019. It sustained the low level until the trough in Jun. 2019, followed by a recovery which led to a peak of the short-cycle growth around Apr. 2021. Also, the following troughs nad crests are shown in [Fig. 7](#page-7-4) (b).
- (2) The predicted medium-to-long-term cycles of Aframax tanker freight rate are 85.3 months (or 7.11 years) and 107.6 months (or 8.97 years), which are primarily embodied in the stage between the short-term cycle and long-term cycle. As shown in [Fig. 7](#page-7-4) (c), the Aframax tanker freight would come into the trough around Dec. 2019 and would come into the next crest around Mar. 2022 according to the first medium-to-long-term cycle. Also, according to the second medium-to-long-term cycle which is shown in [Fig. 7](#page-7-4) (d), the Aframax tanker freight would come into the trough around Jul. 2020 and would come into the next crest around May 2025.
- (3) The predicted long-term cycle of Aframax tanker freight in this paper is 143.5 months (or 11.96 years), Also, the Aframax tanker freight is now at the end of recovery stage and would come into the long-term cycle's trough in around Aug. 2020, and would reach the next crest around Jan. 2027 as showed in [Fig. 7](#page-7-4) (e).

<span id="page-5-0"></span>

Fig. 4. Wavelet Decomposition of Aframax Tanker Freight Through  $dbN (N = 1to6)$ .

<span id="page-5-1"></span>Table 2 Performance of Different dbN Wavelet Functions.

Standard deviation	a1	a2	aЗ
db1	52.99	101.18	155.72
db2	38.26	80.89	111.72
db3	34.97	62.36	124.11
db4	32.30	59.14	100.00
db5	29.73	71.99	116.99
db6	28.17	64.02	101.28

As the Aframax tanker freight's variations were affected by the total influence of all different cycles,.this paper combined the superimposed wavelet coefficient curve with the original Aframax tanker freight curve for analysis. The results are shown in [Fig. 7](#page-7-4) (a). The superimposed WCC is consistent with the peak and trough stage of original Aframax tanker freight curve from 1976 to 2019. The year 2007 saw the subprime mortgage crisis when the amplitudes of various cycles experienced certain degrees of decline, signaling the start of the decline phase of the long-term cycle. In the light of the analysis of the aforementioned cycles on three different timescales' cycles and considering all cycles' influence, the Aframax tanker freight rate is in the medium-to-long-term cycle recession which may continue until around 2020 before it enters the recovery. After the recovery, the growth is expected to continue to 2025. In view of this, the current general trend of the market will continue to face pressure to reduce the excess shipping capacity. As a result, the market fluctuation amplitude is small, presenting a slowly shrinking trend.

<span id="page-6-1"></span>

Fig. 5. Wavelet coefficient map of monthly freight rate of Aframax tankers.

<span id="page-6-2"></span>

Fig. 6. Mean values and standard diviations of different scales' wavelet coefficient curves' extreme Values's interval length.

<span id="page-6-3"></span>



# <span id="page-6-0"></span>5. Conclusion

This paper analyzes the periodic characteristics of the oil tanker shipping market and its influencing factors employing the monthly oneyear time charter rate of Aframax tankers. First, we use wavelet analysis to study the periodic variation laws of tanker freight rate on three timescales. From the analysis, we predicted the corresponding cycle durations on the three time-scales, namely the short-term cycle, the medium-to-long-term cycle, and the long-term cycle. Second, according to the predicted cycles, the paper forecasts the future fluctuation trend of the Aframax tanker market. The following conclusions can be made: (1) The short-term cycle of the market is 41.2 months (or 3.45 years), the medium-to-long-term cycles are 85.3 months (or 7.11 years) and 107.6 months (or 8.97 years), and the long-term cycle is 143.5 months (or 11.96 years); (2) The superimposed wavelet coefficient curve enabled us to know that the Aframax tanker freight rate market entered the recovery of the short-term cycle in around Jun. 2019 and reach the short-term cycle crest in around Apr. 2021. What's more, the Aframax tanker freight rate market comes into the medium-to-long-term cycle trough in around Dec. 2019 to Jul. 2020, and would come into the next

cycles.

crest in around Mar. 2022 to May 2025. Furthermore, the Aframax tanker freight rate market is likely to come into the recovery phase of the long-term cycle from Aug. 2020, and would reach the next crest in around Jan. 2027.

The periodic characteristics of the Aframax tanker freight rate market studied in this paper can offer a reference for tanker market players to learn the future market trends. These periodical features can help entrepreneurs to have a better understanding of the volatility of the Aframax tanker freight rate market, and guide decision-making regarding Aframax tanker market investment and ship purchasing. At the same time, the results of this paper can also help the administrative authority to formulate policies that can better respond to the changes in the tanker timed-charter market. For example, they can roll out tax policies that better answer to the tanker timed-charter market based on the periodic laws, that is, cutting down taxes when the tanker freight rate market is on a decline to encourage the sustained development of the tanker market. Moreover, in view of the influences of the crude oil futures prices on the tanker freight rate, the study on tanker shipping market periodicity can, to some extent, mirror the future trend of the market as crude oil acts as an important energy source for industrial

<span id="page-7-4"></span>

production and its price volatility will inevitably lead to volatility in the entire market.

This paper uses wavelet analysis to study the periodic volatility of the monthly one-year time charter rate of Aframax tankers, and predicts the future cycle variation trend of the market. It draws the macro law of periodic variation in the market. Since the tanker freight rate is subject to the combined influences of periodic and non-periodic factors, we need to carry out further studies on the mechanism that various influencing factors act on the tanker freight rate periodicity and volatility to better learn how the tanker freight rate periodicity and volatility come into being.

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